



Trajectory measurement algorithms

Synchronisation loop algorithm

- Must follow evolution of F_{rev} due to acceleration
- Must reject rapid disturbances

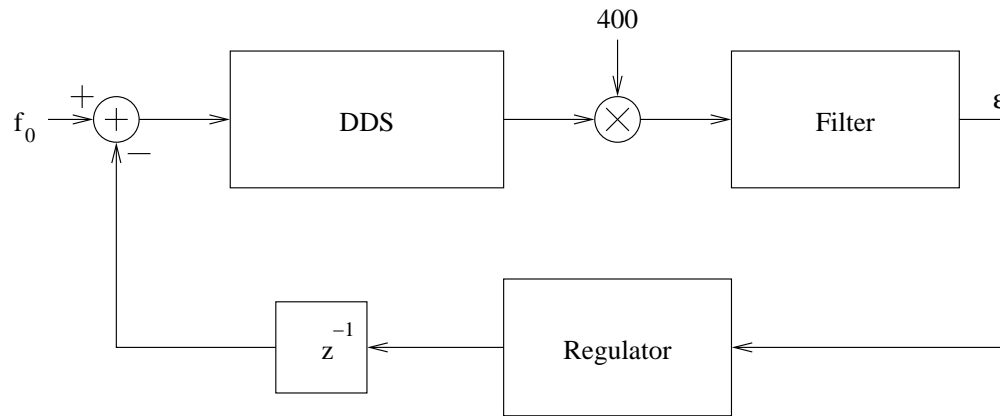
-> Target settling time: 20..200 μ s

-> Target F_{rev} rejection >40dB

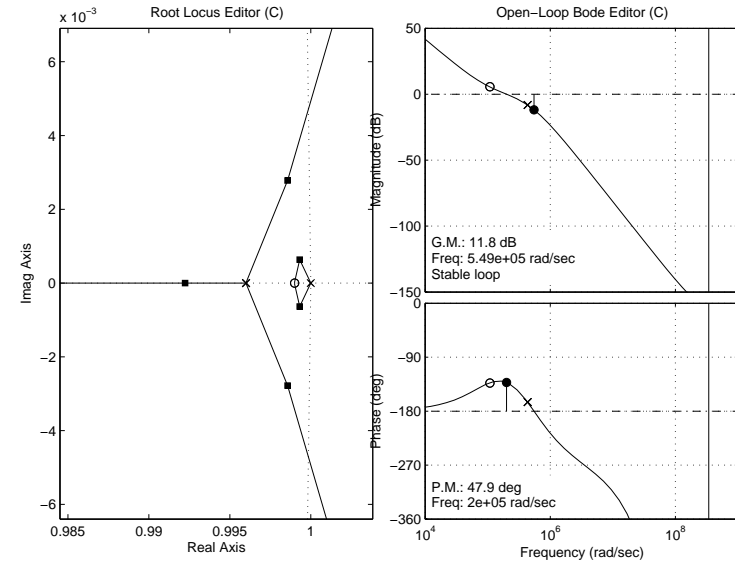


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PLL response analysis



Simplified block diagram



Root locus and Bode plots

DDS

$$H_{dds} = \frac{h}{2^{32}} \frac{z^{-1}}{1-z^{-1}}$$

Mixer

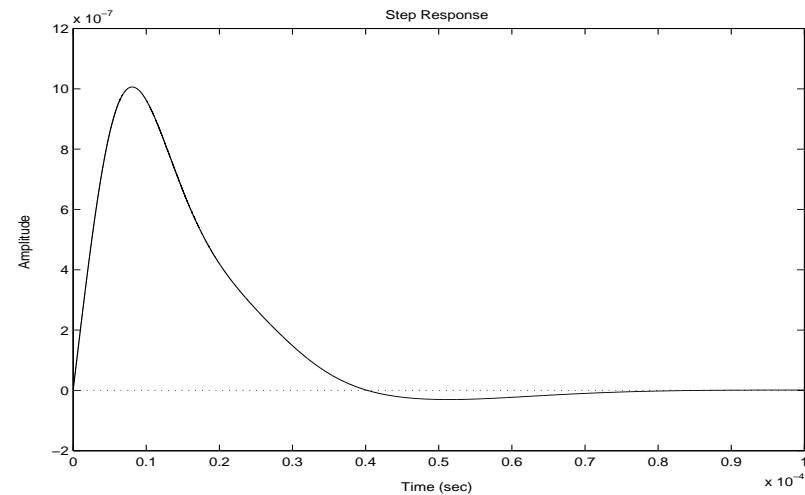
$$H_m = 400$$

Low-pass filter

$$H_F = \frac{1}{256} \left\{ \frac{z^{-1}}{1-0.996z^{-1}} \right\}^3$$

Regulator

$$H_R = K_R \cdot z^{-3} \cdot \frac{(1-0.999z^{-1})^2}{(1-z^{-1})}$$



Step (in ϵ) response



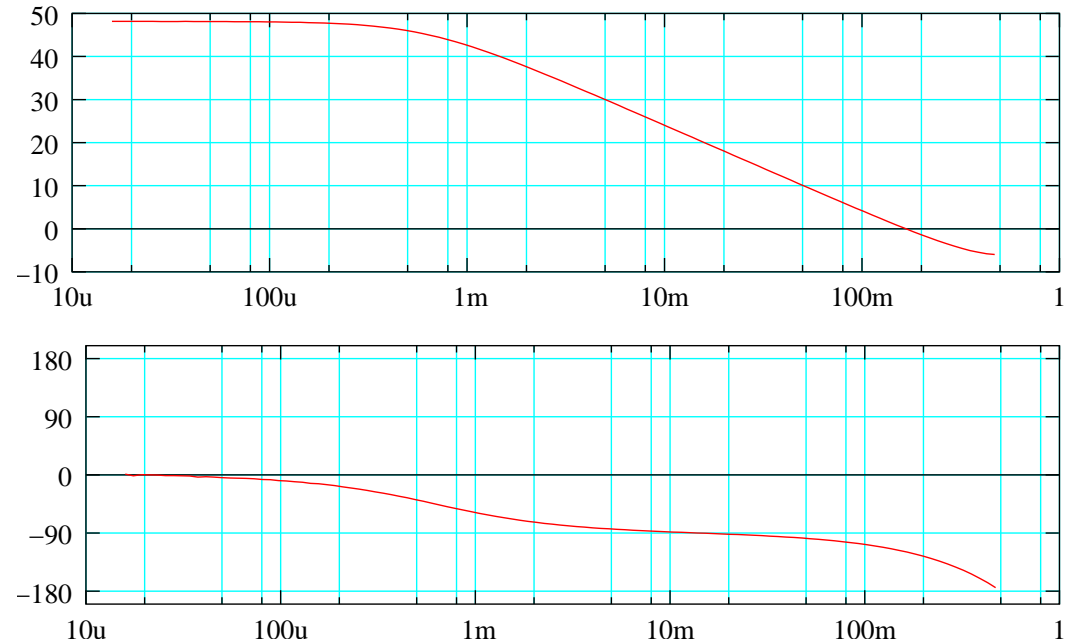
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Low pass filter stage

$$\frac{y}{x} = \frac{z^{-1}}{1 - 0.996z^{-1}}$$

$$y_n = 0.996y_{n-1} + x_n$$

$$y = x + y - (y \gg 8);$$



Bode plot



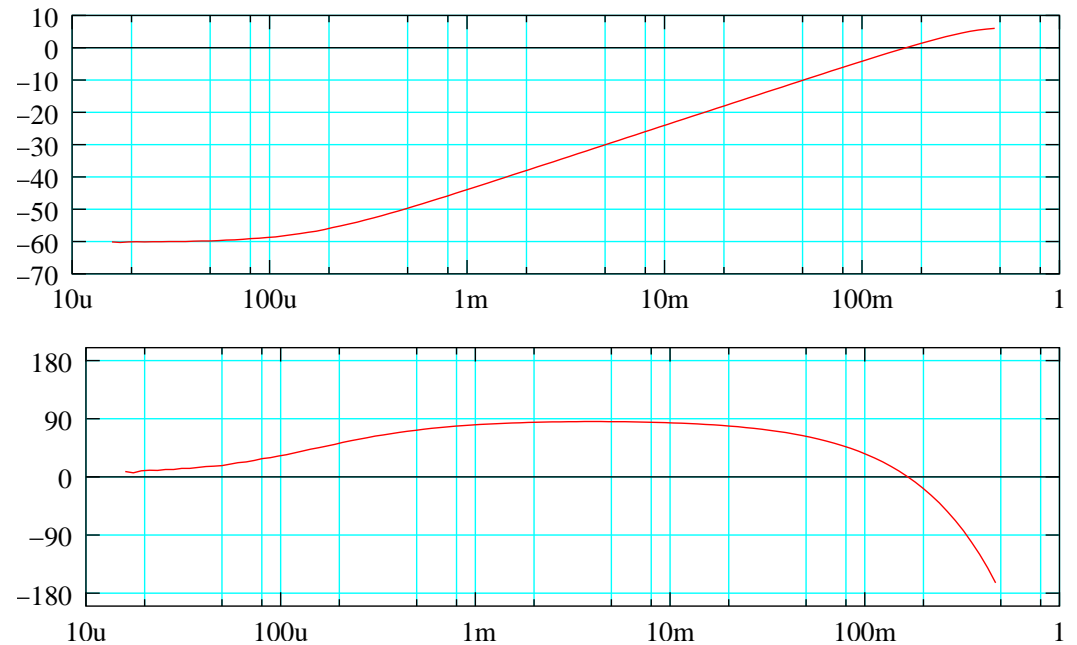
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High pass filter stage

$$\frac{y}{x} = z^{-1}(1 - 0.999z^{-1})$$

$$y_n = x_n - 0.999x_{n-1}$$

$$y = x - ix;$$
$$ix = x - (x \gg 10);$$



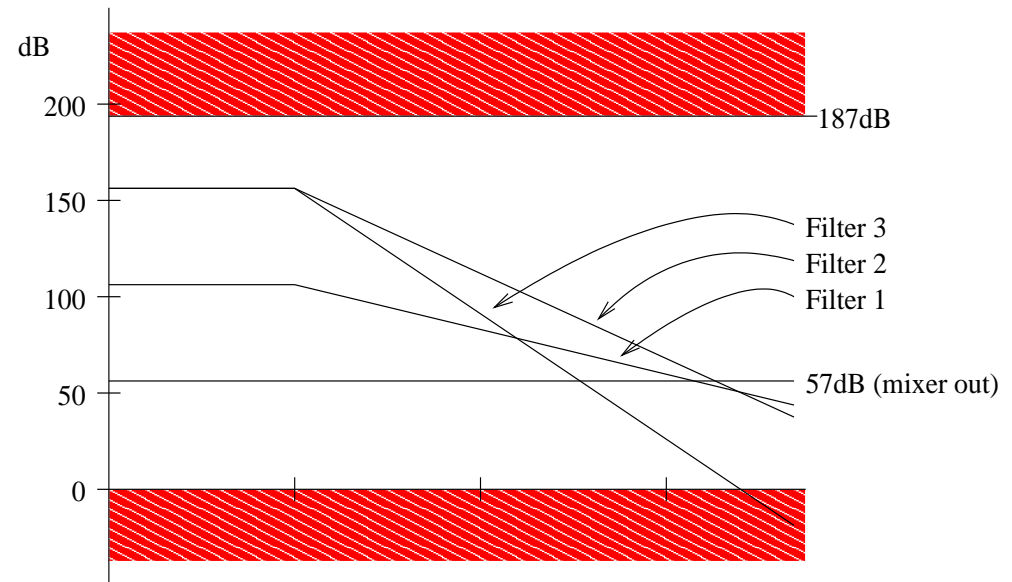
Bode plot



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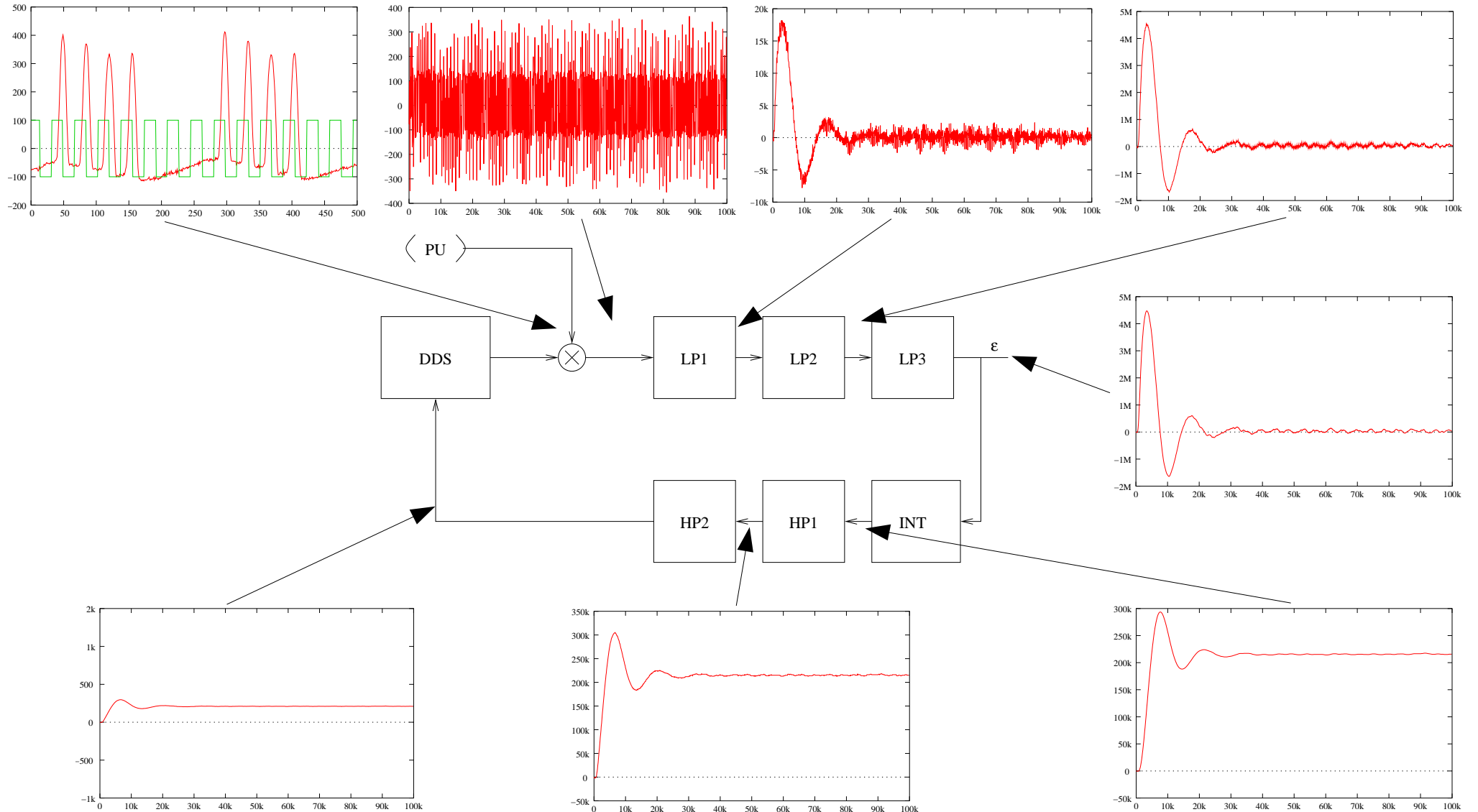
A 32 bit integer has 187dB dynamic range

- Scale each stage
- Compensate loop gain





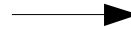
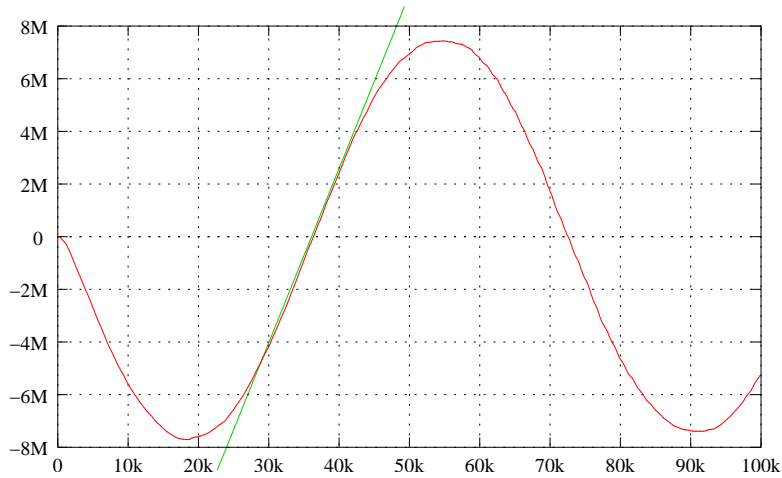
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Phase noise I



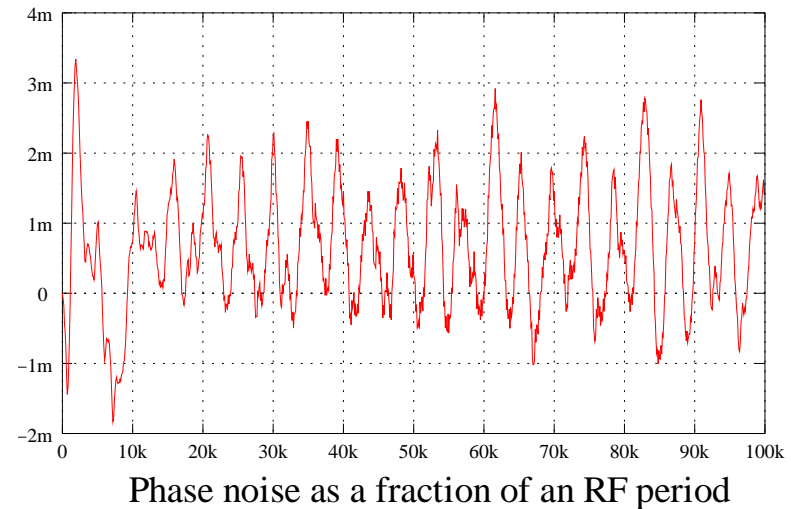
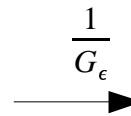
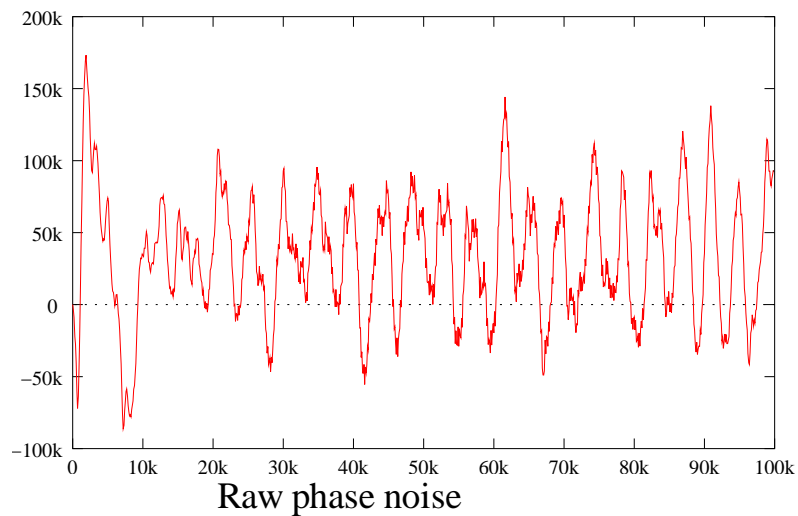
First find the phase error gain:

$$f(x) = 8M \sin \frac{2\pi x}{72k}$$

$$f'(x) = \frac{16\pi M}{72k} \cos \frac{2\pi x}{72k}$$

Normalised to one period of the phase error:

$$G_\epsilon = 16\pi M = 50 \cdot 10^6$$

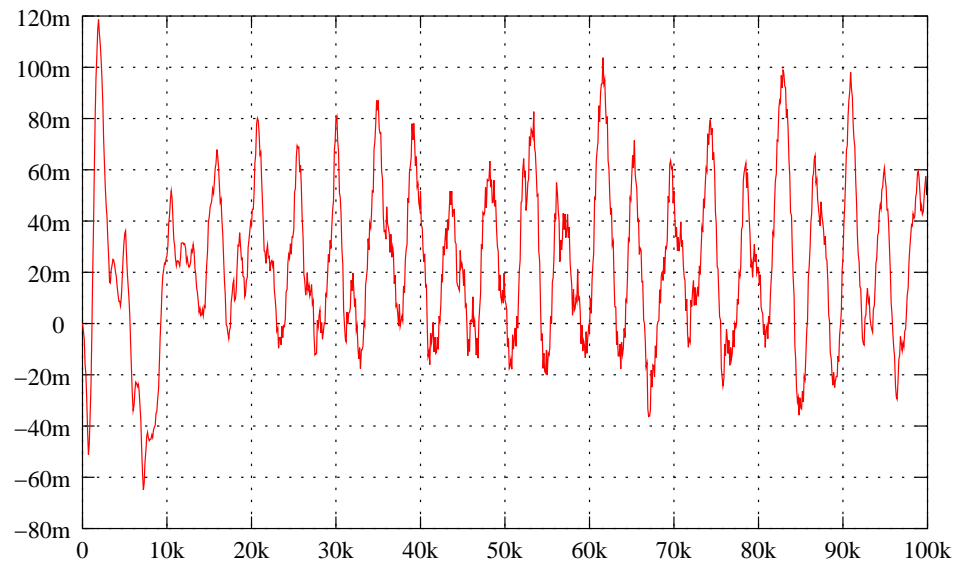




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Phase noise II

There are $\frac{2^{32}}{17296830h} = 35.5$ samples in one RF period. (For LHC, $h=7$)

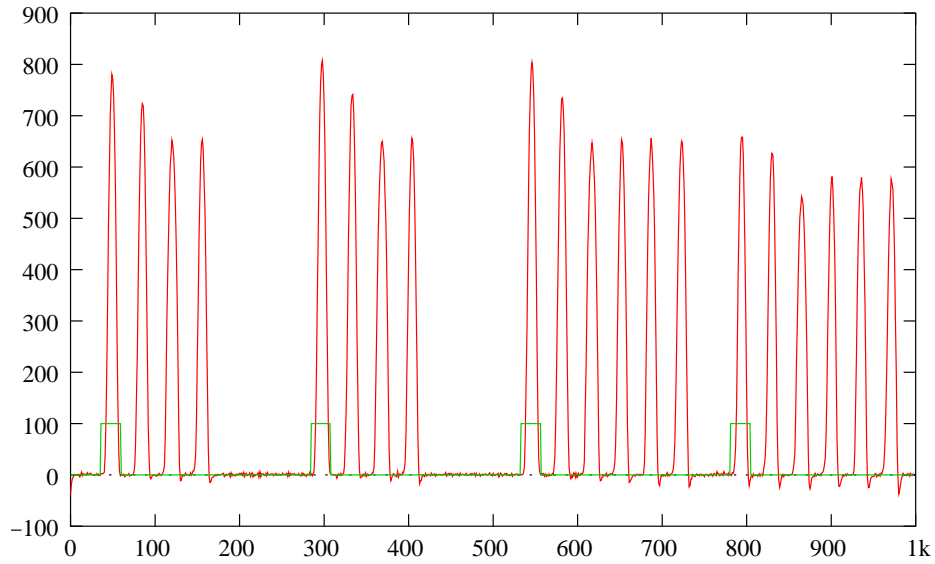


Phase noise normalised to one sample

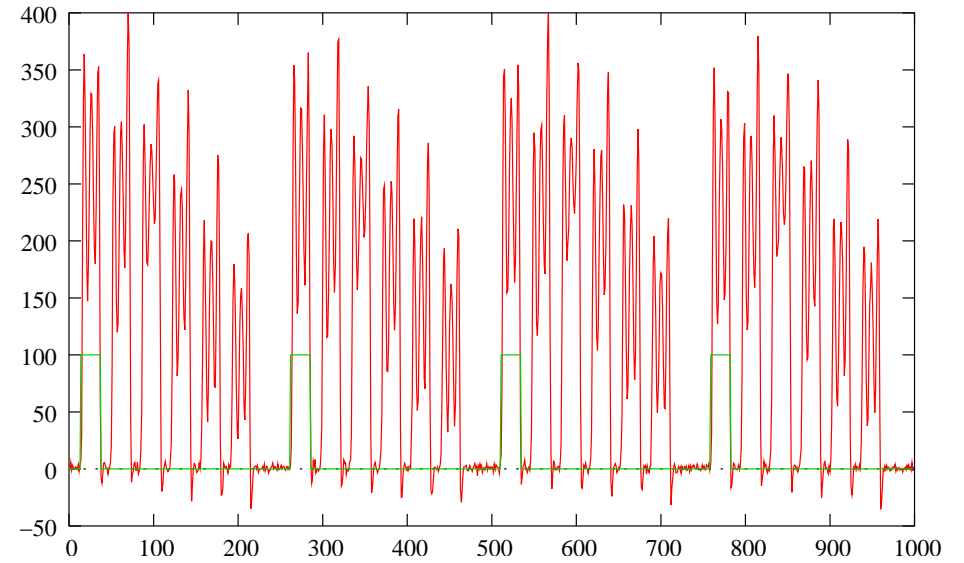
$\sigma=0.03$ samples



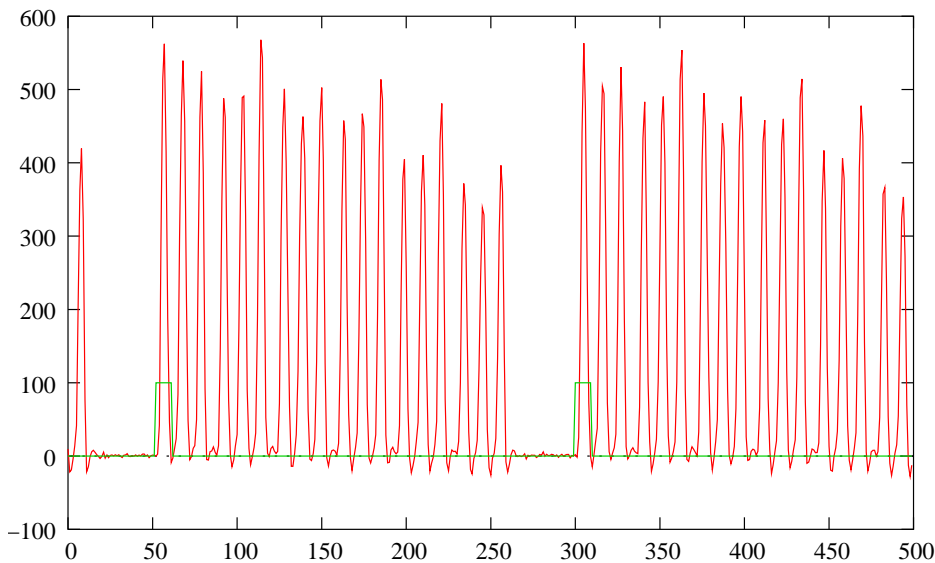
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Example of generated gate around 2nd injection



Idem, during bunch splitting



And after the harmonic change